

Fractions

Question 1

- a) How many rectangles are there ?
 b) How many are shaded ?
 c) Express this as a fraction
 d) How many are not **shaded**?
 e) Express **unshaded** as a fraction

Solution

- a) There are 10 rectangle spaces.
 b) 3 spaces are shaded
 c) $\frac{\text{Number..shaded}}{\text{Total..number..of..spaces}} = \frac{3}{10}$
 d) 7 remaining are unshaded
 e) $\frac{7..remaining..unshaded..}{\text{Total..number..of..spaces}} = \frac{7}{10}$

A fraction consists of a upper numerator and a lower denominator. When the **denominators** are the **same** we can **add or subtract** the **numerators**

Question 2

- a) In the fraction $\frac{7}{10}$ state the numerator and denominator respectively
 b) add $\frac{7}{10} + \frac{2}{10}$
 c) Subtract $\frac{7}{10} - \frac{1}{10}$

Solutions

- a) In the fraction $\frac{7}{10}$; 7 is the numerator , and 10 is the denominator.

b) $\frac{7}{10} + \frac{2}{10}$ as denominators are the same we add the numerators $\frac{7+2}{10} = \frac{9}{10}$

c) $\frac{7}{10} - \frac{1}{10}$ as denominators are the same we add the numerators $\frac{7-1}{10} = \frac{6}{10}$..cancel..down.. = $\frac{3}{10}$

Did you notice that we divided **both** numerator & denominator by 2 ?

Exercises 1

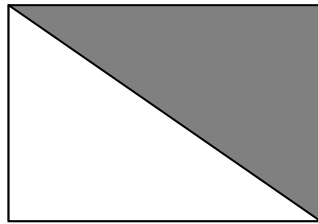
1)
Express as fraction a) The shaded region b) The unshaded region

2)
Add a) $\frac{2}{11} + \frac{4}{11}$ b) $\frac{1}{12} + \frac{5}{12}$ c) $\frac{2}{9} + \frac{4}{9}$

Equivalent fractions

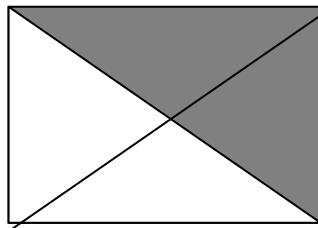
Here we have shaded only **one half** ($\frac{1}{2}$) in fig: a

Fig: a



We now divide the same fig into 4 equal parts to give us fig: b

Fig b



both figures a&b

We have shaded 2 out of the 4 parts ie $\frac{2}{4}$
How ever the **same regions** have been shaded in

Hence $\frac{1}{2} = \frac{2}{4}$ These 2 fractions are called **equivalent fractions**

Types of fractions

- **Proper fraction** Here numerator is less than denominator eg $\frac{2}{7}$
- **Improper fraction** Here numerator is greater than denominator eg $\frac{11}{7}$
- **Mixed number** Here one whole number and a proper fraction $3\frac{2}{5}$

To convert a **fraction** into its **equivalent** form we must **multiply** both **numerator** & **denominator** by the **same number**

$\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$ Here we have multiplied **both** numerator & denominator by **2**

$$\frac{1}{2} = \frac{2}{4}$$

Question 3

By what numbers each should you multiply both numerator & denominator of fraction $\frac{2}{5}$

To give a) $\frac{6}{15}$ b) $\frac{14}{35}$ c) $\frac{16}{40}$

Solution

a) We should try to convert the denominator 5 into a 15, hence we multiply num:&den: by 3

$$\frac{2 \times 3}{5 \times 3} = \frac{6}{15}$$

b) We should try to convert the denominator 5 into a 35 hence we multiply num:&den by 7

$$\frac{2 \times 7}{5 \times 7} = \frac{14}{35}$$

c) We should try to convert the denominator 5 into a 40 hence we multiply num:&den by 8

$$\frac{2 \times 8}{5 \times 8} = \frac{16}{40}$$

Exercise 2

1) Fill in the frame to give a equivalent fraction

$$a) \frac{9}{11} = \frac{[\]}{44} = \frac{72}{[\]}$$

$$b) \frac{5}{6} = \frac{45}{[\]} = \frac{[\]}{72}$$

$$c) \frac{8}{9} = \frac{96}{[\]} = \frac{[\]}{63}$$

a) State which fraction is equivalent to

$$a) \frac{1}{3} = \frac{3}{4} = \frac{2}{8} = \frac{4}{12} = \frac{10}{33}$$

$$b) \frac{4}{5} = \frac{12}{20} = \frac{44}{48} = \frac{24}{30} = \frac{22}{25}$$

Simplest Form

Here we divide both denominator & numerator by the Highest Common Factor (HCF) to give the fraction in its simplest form, also called **cancelling down**

Question

Find the highest common factor of 6,18

Solution

Factors of 6 1 2 3 6

Factors of 12 1,2,3,6,9,12

Answer even though 1,2,3,6 are common to both **6 is the highest of them all** hence HCF is **6**

Question

Find the HCF and cancel down the following fractions a) $\frac{6}{18}$ b) $\frac{45}{75}$ c) $1\frac{9}{12}$

a) Factors of 6 1 2 3 6

Factors of 12 1,2,3,6,9,12 Answer: HCF is **6**. Divide both num: and den: by **6**

$$\text{Hence } \frac{6 \div 6}{18 \div 6} = \frac{1}{3}$$

b) Factors of 45 1, 3, 5, 9 15, 45

Factors of 75 1,3,5, 15, 25, 75 Answer HCF is **15** Divide both num: and den: by **15**

$$\text{Hence } \frac{45 \div 15}{75 \div 15} = \frac{3}{5}$$

c) Mixed number $1\frac{9}{12}$

Factors of 9 1,3,9

Factors of 12 1,2,3,4,6,12 Answer HCF is **3** Divide both num: and den: by **3**

$$1 \frac{9 \div 3}{12 \div 3} = 1 \frac{3}{4}$$

Simplify by cancelling down

1) a) $\frac{25}{55}$ b) $\frac{63}{77}$ c) $\frac{72}{81}$

2) a) $\frac{96}{144}$

3) a) $3 \frac{21}{63}$ b) $7 \frac{18}{32}$

Convert improper fraction to mixed number

In a **improper fraction**, num: is greater than den: eg $\frac{8}{3}$



+



+

so we have 2 wholes and $\frac{2}{3}$ of a whole



- We divide the num: by the den:
- Then write the **remainder** as a fraction eg as $\frac{2}{3}$
- $\frac{8}{3} = 2 + \frac{2}{3} = 2\frac{2}{3}$

Question

Convert the **improper fraction** into a mixed number

a) $\frac{49}{12}$ b) $\frac{56}{9}$

Solution

$$a) \frac{49}{12} = 4 + \frac{1}{12} = 4\frac{1}{12}$$

$$4 \times 12 = 48 \text{ and remainder is } 1$$

$$b) \frac{56}{9} = 6 + \frac{2}{9} = 6\frac{2}{9}$$

$$6 \times 9 = 54 \text{ and remainder } 2$$

Convert mixed number into a improper fraction

- Denominator x whole number
- Then add result to numerator
- Express result as a improper fraction

Question

Write the mixed number $4\frac{1}{12}$ as a improper fraction

Solution

Whole = 4 , numerator = 1

$$\frac{4 \times 12 + 1}{12} \quad \text{ie} \quad \frac{48 + 1}{12} = \frac{49}{12} \dots\dots\dots \text{Improper fraction}$$

Exercise

1)

Convert the mixed number into a improper fraction

$$a) 2\frac{3}{4} \qquad b) 12\frac{1}{12} \qquad c) 9\frac{3}{8}$$

2)

Convert the improper fraction $\frac{23}{7}$ into a mixed number by stating the

- a) whole number b) The fraction portion c) The entire mixed number

3)

There were similar pies on a table and Jane ate two and a quarter pies

a) express this as a mixed number b) Hence as improper fraction

Additions and subtraction of fractions

- When denominators are the same we add or subtract the numerators
- Convert each fraction to one with same denominator using the law of equivalent fractions
- we can add or subtract the numerators as the denominators are the same

Question

Add (When one denominator is a factor of the other)

1)

a) $\frac{1}{7} + \frac{3}{7}$ b) $\frac{2}{5} + \frac{3}{10}$ c) $\frac{1}{7} + \frac{3}{14}$ d) $\frac{2}{9} + \frac{4}{27}$

Solutions

1)

a) Here denominators are the same so we just **add the numerators**

$$\frac{1}{7} + \frac{3}{7} = \frac{1+3}{7} = \frac{4}{7}$$

b) Here denominators are **different** so we must use the law of equivalent fractions to make the **denominators the same** by converting 5 into 10 ie $5 \times 2 = 10$, Hence **10** is our common den:

$$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10} \text{ hence the equivalent fraction of } \frac{2}{5} \text{ is } \frac{4}{10}$$

We can now add $\frac{4}{10} + \frac{3}{10} = \frac{7}{10}$

We have multiplied both
Num & Den : by **2** of the
fraction $\frac{2}{5}$

c) Denominators are **different** so we use equivalent fractions to make them the **same (14)**

$$\frac{1}{7} + \frac{3}{14} = \frac{1 \times 2}{7 \times 2} + \frac{3}{14} = \frac{2}{14} + \frac{3}{14} = \frac{2+3}{14} = \frac{5}{14}$$

We have multiplied both
Num & Den : by **2** of the
fraction $\frac{1}{7}$

d) Denominators are **different** so we use equivalent fractions to make them the **same (27)**

$$\frac{2}{9} + \frac{4}{27} = \frac{2 \times 3}{9 \times 3} = \frac{6}{27} \quad \text{Now we add} \quad \frac{6}{27} + \frac{4}{27} = \frac{6+4}{27} = \frac{10}{27}$$

We have multiplied both
Num & Den : by **2** of the
fraction $\frac{1}{7}$

2)

Subtraction (**When one denominator is a factor of the other denominator**)

$$\text{a) } \frac{3}{7} - \frac{1}{7} \qquad \text{b) } \frac{3}{5} - \frac{2}{10} \qquad \text{c) } \frac{3}{7} - \frac{1}{14}$$

Solution

$$\text{a) } \frac{3}{7} - \frac{1}{7} \quad \text{Here the denominators are the same hence we just **subtract** the numerators}$$

$$\frac{3-1}{7} = \frac{2}{7}$$

$$\text{b) } \frac{3}{5} - \frac{2}{10} \quad \text{As the denominators are **different** we use equivalent fractions to make them the **same**}$$

$$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10} \quad \text{Hence } \frac{4}{10} - \frac{2}{10} = \frac{2}{10} \quad \text{We can cancel down ie divide both den: & num: by 2}$$

$$\frac{2 \div 2}{10 \div 2} = \frac{1}{5}$$

$$\text{c) } \frac{3}{7} = \frac{3 \times 2}{7 \times 2} = \frac{6}{14} \qquad \frac{6}{14} - \frac{1}{14} = \frac{5}{14}$$

Note Denominators
are the **same**

Exercises

Add 1) a) $\frac{2}{11} + \frac{3}{44}$ b) $\frac{5}{7} + \frac{11}{42}$ c) $\frac{8}{63} + \frac{5}{9}$

Subtract 2) a) $\frac{5}{9} - \frac{3}{45}$ b) $\frac{7}{12} - \frac{11}{96}$ c) $\frac{9}{10} - \frac{3}{50}$

Addition by finding the lowest common denominator (LCD)

- Obtain the **lowest common denominator**
- Use equivalent fractions to get a common denominator
- Now **add** numerators

Question**Add**

a) $\frac{2}{7} + \frac{4}{5}$ b) $\frac{1}{3} + \frac{3}{4}$ c) $1\frac{5}{7} + 2\frac{1}{6}$

Solutions

a) Common multiples of 7 are

7, 14, 21, 28, 35,

Common multiples of 5 are

5, 10, 15, 20, 25, 30, 35

35 is common to both

$$\frac{2 \times 5}{7 \times 5} = \frac{10}{35}$$

$$\frac{4 \times 7}{5 \times 7} = \frac{28}{35}$$

$$\frac{10}{35} + \frac{28}{35} = \frac{10+28}{35} = \frac{38}{35} = 1\frac{3}{35}$$

b) Common multiples of 3 are

3, 6, 9, 12

Common multiples of 4 are

4, 8, 12

12 is common to both

$$\frac{1 \times 4}{3 \times 4} = \frac{4}{12}$$

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

$$\frac{4}{12} + \frac{9}{12} = \frac{4+9}{12} = \frac{13}{12} = 1\frac{1}{12}$$

c)

- Convert mixed number into a improper fraction

$$2\frac{5}{7} + 5\frac{1}{6} = \frac{19}{7} + \frac{31}{6} \quad \text{improper fraction}$$

42 is common
to both

Multiples of **7** are 7,14,21,28,35,**42**.

Multiples of **6** are 6,12,18,24,30,36,**42**

- Convert to fractions with same denominator using equivalent fractions

$$\frac{19 \times 6}{7 \times 6} = \frac{114}{42} \quad \longrightarrow \quad \frac{114}{42} + \frac{217}{42}$$

$$\frac{31 \times 7}{6 \times 7} = \frac{217}{42}$$

- Add the numerators as denominators are now the same

$$\frac{114}{42} + \frac{217}{42} = \frac{217 + 114}{42} = \frac{331}{42}$$

- convert into a mixed number by division

$$\frac{331}{42} = 42 \overline{)331}^7 = 7 \frac{331 - 297}{42} = 7 \frac{37}{42}$$